Survey of India Department.

PROFESSIONAL PAPER—No. 1.

ON THE PROJECTION

FOR A MAP

OF

INDIA AND ADJACENT COUNTRIES

On the Scale of 1 : 1,000,000

BY

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PREPARED UNDER THE DIRECTION OF

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The extent of country which it is proposed that the new map should embrace, reaches longitudinally from the western frontier of Persia to the extreme east of Burma, that is, from 44° to 104° east longitude, and in latitude from the south of Ceylon to the Oxus or from 4° to 40° north latitude.

The scale of the map, $\frac{1}{1,000,000}$, has been chosen to fall in with the views expressed by the International Geographical Congress, who have proposed that a map of the world be undertaken on that scale.

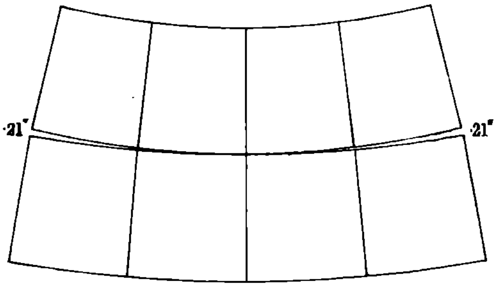
A general Map of India and Adjacent Countries on the scale of 16 miles to one inch has long been a desideratum, and the scale of the proposed map, *viz.*, $\frac{1}{1,000,000}$, being equivalent to one of 15·78 miles to one inch, is so very close to that of the map required that it may well be substituted for it.

The system of projection to be proposed by the International Geographical Congress has not yet been decided upon, and as it is inadvisable to delay the commencement of the map, it has become necessary to select a suitable projection.

2. Broadly speaking there are two systems of projection which may be adopted for such a map; one is that in which each sheet is projected on its own central meridian and the other is to have one projection for the whole map.

The first method has the great advantage that by its use each sheet can be made practically true to scale and free from distortion. Examples of projections suitable for this method are the ordinary Survey of India projection used for standard sheets and the Rectangular Tangential projection of Sir H. James, employed by the Home War Office. Over moderate areas these two projections may be regarded as identical.

The disadvantage of this method is that it is impossible to join any number of sheets together to make a large map. Thus in the Rectangular Tangential projection if it is attempted to join up 8 sheets $4^\circ \times 5^\circ$, of a $\frac{1}{1,000,000}$ map together on their central meridian as in the sketch, the two belts will separate at each end on the central parallel to the extent of ·21 inch. The Survey graticule gives about the same amount of misfit.



3. As it has been considered essential in the case of the present map that all the sheets should fit together exactly to form one map, the second of the above-mentioned methods must be adopted, that is there must be one projection for the whole map.

As the area to be included is a very large portion of the globe, embracing 36° of latitude and 60° of longitude, some form of conical projection is clearly indicated.

4. In order to keep distances along the two co-ordinates true to scale, a system such as Bonne's modification of the conical projection suggests itself; but the area is so extensive that it is found that if this projection be adopted the distortion of the corner sheets becomes very marked.

5. As the map embraces 36° of latitude, a simple conical projection on any central parallel, while meeting satisfactorily the difficulties of the great longitudinal expanse, falsifies the scale very largely in longitude along the north and south limits of the map.

To avoid this, as far as possible, it seems best to adopt a conical projection where the cone is a secant at two selected parallels of latitude. This will minimize the scale error to a considerable extent. A certain amount of error is of course unavoidable; this takes the form of all latitudinal distances, north and south, being slightly too short, while longitudinal distances measured along the parallels are correct on the selected secant parallels, too short between these parallels and too long beyond their limits north and south. The error in longitude between the selected parallels mentioned in the next paragraph is never so much as 1 per cent, that on the extreme northern margin of the map about 3 per cent which is about the same as that on the parallel of 8°, while the error on the extreme southern parallel of 4° is nearly 5 per cent.

6. However the whole amount of error introduced is not of much importance in a map of this class, which is on a scale smaller than that employed in any actual field surveys and it has been decided therefore to adopt a secant conical projection, the cone cutting the parallels of 16° and 32°.

7. The longitudes hitherto given on all maps published in India have been based on one or other of the old value of that of Madras Observatory, *viz.*, 80° 18′ 30″ for the sheets of the Indian Atlas and 80° 17′ 21″ for standard sheets and general maps.

As the longitude of Madras has now been ascertained within limits of error quite inappreciable on topographical maps and as the map under consideration embraces countries outside India in which our surveys join up with those of other countries, it is advisable to eliminate the error in longitude.

The longitudes therefore of the Map of India and Adjacent Countries will be referrible to the Greenwich meridian, taking that of Madras Observatory as 80° 14′ 47″, the most recently determined value.

8. The question of the size and shape of the sheets requires some consideration.

The borders of the sheets of the Indian Atlas are rectangular and projected on one central meridian for the whole atlas. They thus fit accurately together. On the other hand the sheets are very difficult to plot, and most difficult to use for measuring off latitudes and longitudes or for compiling from. Moreover even within the comparatively narrow limits of longitude embraced by the Indian Atlas the flank sheets are much disfigured by the graticule and names being printed at a considerable angle with the margin. This tilting of names is of little importance when the whole extent of the map is contained in one or two sheets as in hand atlases, as the eye follows the curves of the names along the parallels; but in larger scale maps when one sheet only contains a small portion of the map, the disfigurement is very noticeable. In the case of the map under consideration where 60° of longitude are embraced, this defect would be painfully apparent in the outer sheets.

The alternative shape for the sheets is that so long used throughout the Survey Department, where the bounding lines of the sheet are parallels of latitude and longitude. These would give the side margins of the sheet as straight but slightly converging lines, and the top and bottom margins concentric curves. Beyond a very slight increase of trouble in cutting the top and bottom margins whenever it may be necessary to fit two sheets together, this system presents no drawback of any moment, and it is therefore decided to adopt it. The area embraced by each sheet will be 4° of latitude and 4° of longitude.

9. The sheets of India proper will be engraved; those of the outlying portions, Persia, Afghanistan &c., will be photozincographed, at least in the first instance.

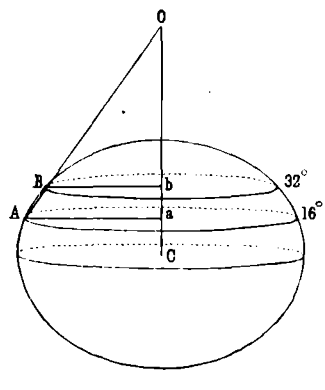
A border will be designed which will be common to both sets, so as to make the general appearance of the sheets as far as may be similar.

10. It should be noted that in selecting this particular projection for the map, it is in no wise intended to put it forward as an improvement on or as superseding the present Survey Projection as used on standard maps. It is merely considered to fulfil best the conditions which were required to be met in the case of the map under discussion, the chief of which is the correct fitting together of the sheets.

The Secant Conical Projection.

The projection used in the preparation of this map is a secant conical one in which the cutting cone passes through the parallels of 16° and 32°.

Let OBA be the generator of the cone which lies in the plane of the paper and cuts the earth in B and A latitudes 32° and 16°. Let C be the centre of the ellipse in the plane of the paper, and let a and b be the points where the perpendiculars from A and B to the minor axis meet the latter.



Then if ν is the normal terminated by the minor axis in latitude λ and if $[\nu \cos \lambda]_A$ represents the value of $\nu \cos \lambda$ at the latitude of A, we have by well-known properties of the ellipse, using Everest's constants

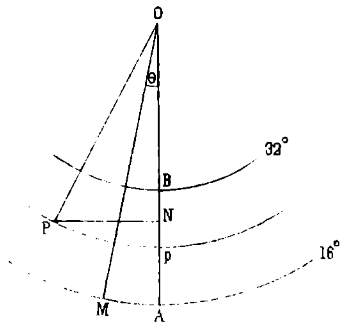
$$Aa = [\nu \cos\lambda]_A = 20117486 \text{ feet} \quad \text{and} \quad Ca = (1 - e^2)\ [\nu \sin\lambda]_A = 5730305 \text{ feet}$$

$$Bb = [\nu \cos\lambda]_B = 17760213 \text{ feet} \quad \text{and} \quad Cb = (1 - e^2)\ [\nu \sin\lambda]_B = 11024147 \text{ feet}$$

whence
$$AB = \sqrt{(Aa - Bb)^2 + (Cb - Ca)^2} = 5794959 \quad \text{feet}$$

and
$$OA = \quad Aa \cdot \frac{AB}{Aa - Bb} \quad = 49455489 \text{ feet.}$$

Now take a point P which differs from A by $\Delta\lambda$ in latitude and ΔL in longitude. Let θ be the angle subtended at O by 1° of longitude



then
$$\frac{AM}{Aa} = \text{circular measure of } 1°$$

$$\frac{AM}{OA} = \quad \text{,,} \quad \text{,,} \quad \theta$$

therefore
$$\theta \text{ in degrees} = \frac{Aa}{OA}$$

and
$$\angle POp = \Delta L \cdot \theta.$$

Now in this system of projection the length AB is divided into sixteen equal parts and each represents one degree on the meridian.

Thus
$$OP = OA - Ap = OA - \Delta\lambda \cdot \frac{AB}{16}$$

whence the co-ordinates of P referred to the axes AO and a line through A perpendicular to AO are

$$x = PN = OP \sin \angle POp$$

$$y = AN = OA - OP \cos \angle POp.$$

For convenience of plotting, the origin is now changed from the point A to the centre of the lower parallel forming the southern margin of each sheet and the axes are changed to the line joining this to the point O and a line perpendicular to it. As the co-ordinates referred to such origins are the same for all sheets lying between the same parallels, it is only necessary to find them for the sheet adjacent to OA.

We have

x and y the co-ordinates of P with respect to Ax and Ay

and if

X and Y be the co-ordinates of P with respect to ωX and ωY

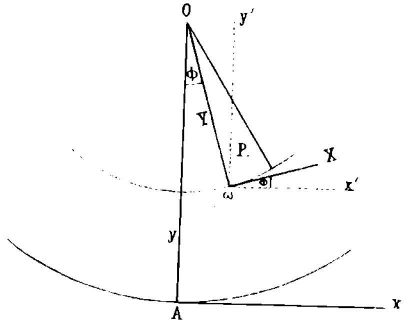
x' and y' ,, ,, P ,, ,, $\omega x'$ and $\omega y'$ parallel to Ax and Ay

and

ζ and η ,, ,, ω ,, ,, Ax and Ay

and $\phi =$ the angle ωOA

then by the usual formulæ



$$X = x' \cos \phi + y' \sin \phi$$

$$Y = y' \cos \phi - x' \sin \phi$$

and

$$x = x' + \zeta$$

$$y = y' + \eta$$

Whence

$$X = (x - \zeta) \cos \phi + (y - \eta) \sin \phi$$

$$Y = (y - \eta) \cos \phi - (x - \zeta) \sin \phi$$

and as each sheet has a breadth of 4° in longitude, ϕ is the value of θ for 2° of longitude.

MAP OF INDIA AND ADJACENT COUNTRIES.

Rectangular Co-ordinates for plotting the Graticules of the Sheets, Scale 1 : 1,000,000.

Latitude	Meridians from Origin			
	0°		± 2° East or West	
	Meridian	Perpendicular	Meridian	Perpendicular
	Inches	*Inches*	*Inches*	*Inches*
SHEETS ... { 40° ...	17·385	0	17·435	6·945
36° ...	0	0	0·051	7·194
" ... { 36° ...	17·385	0	17·436	7·194
32° ...	0	0	0·054	7·440
" ... { 32° ...	17·385	0	17·439	7·440
28° ...	0	0	0·054	7·687
" ... { 28° ...	17·385	0	17·439	7·687
24° ...	0	0	0·056	7·934
" ... { 24° ...	17·385	0	17·441	7·934
20° ...	0	0	0·058	8·181
" ... { 20° ...	17·385	0	17·443	8·181
16° ...	0	0	0·060	8·427
" ... { 16° ...	17·385	0	17·445	8·427
12° ...	0	0	0·062	8·675
" ... { 12° ...	17·385	0	17·447	8·675
8° ...	0	0	0·064	8·921
" ... { 8° ...	17·385	0	17·449	8·921
4° ...	0	0	0·065	9·168

Directions for Plotting.—A point on the sheet is selected for the intersection of the central meridian and the lower parallel. Through this point a horizontal line is drawn across the sheet, and from the same point a second line perpendicular to the first: the second line will be the central meridian of the sheet. With these two lines as axes the intersections of the meridians and parallels are plotted from this table of rectangular co-ordinates in the usual way. As the sheet is symmetrical about its central meridian, the values for 2° are to be used for intersections both east and west of that meridian.

The exterior meridians and parallels are formed by joining these points by straight lines and the central parallel is obtained by bisecting the meridians and joining up the points.

ST. G. C. GORE, COLONEL, R.E.

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INDIA & ADJACENT COUNTRIES

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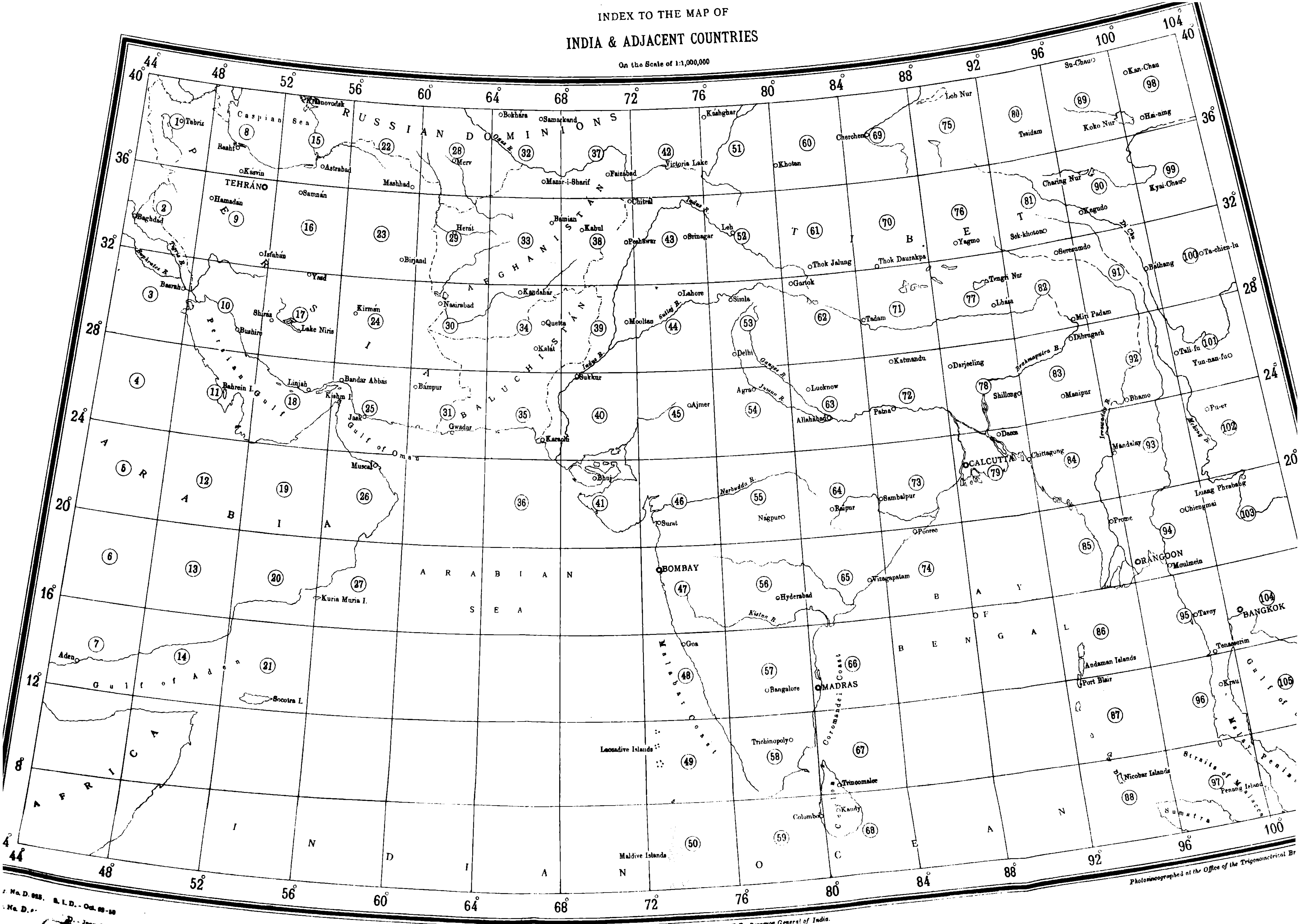

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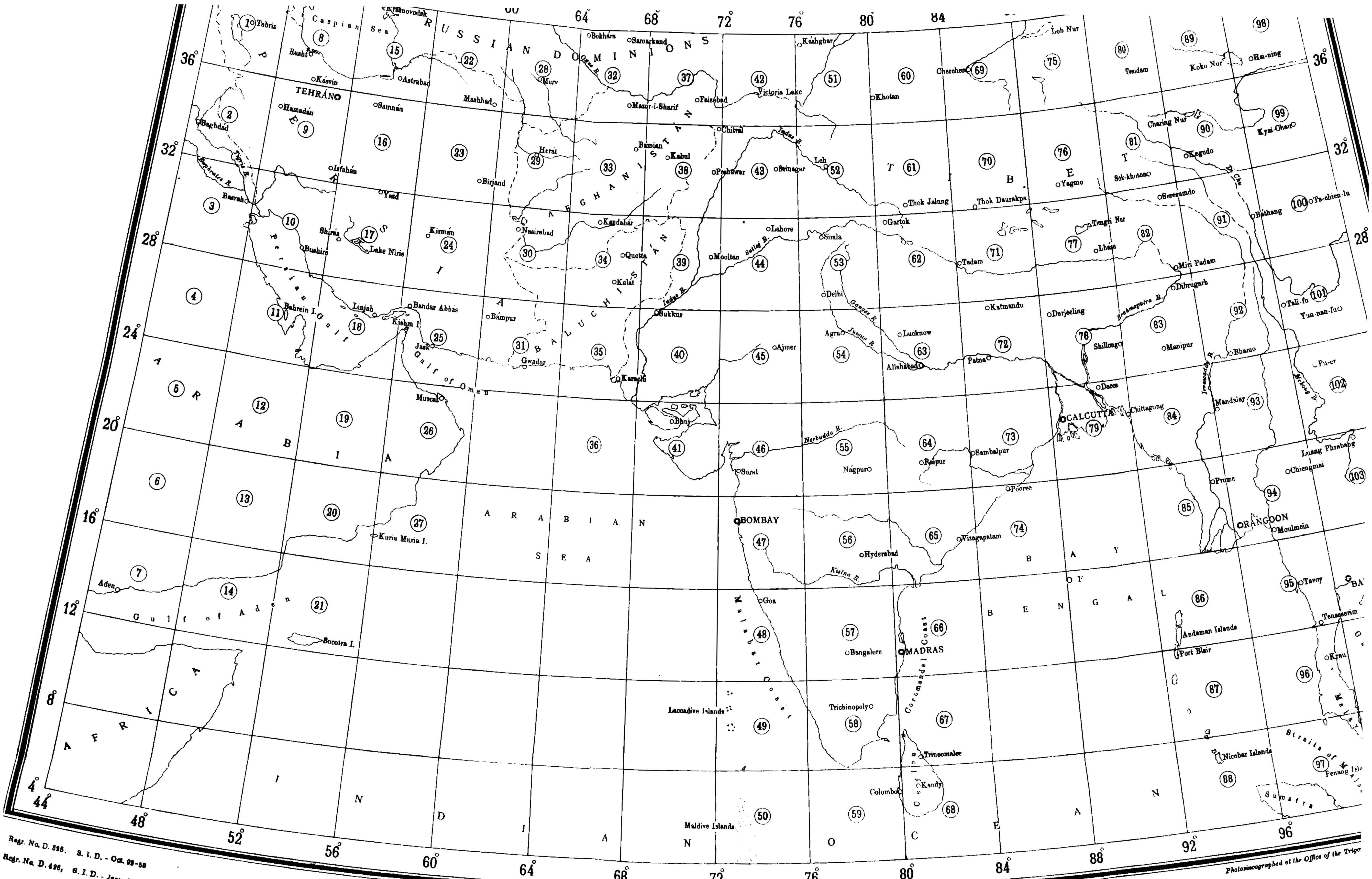
Scale of Miles



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100 50 0 100 200 300 400

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